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FEM analysis of multifluid heat exchangers

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Abstract Heat exchangers are devices for exchanging energy between two or more fluids. They find applications in various industries like power, process, electronics, refining, cryogenics, chemicals, metals and manufacturing sector. Even though heat exchanger designs have been reported quite extensively, they are generally limited to steady-state performance, single phase fluids, a few of the many possible flow arrangements and only two fluid heat exchangers. While these designs encompass the majority of the heat exchanger applications, there are some designs, which involve several fluids such as in cryogenics or fault-tolerant heat exchangers. The governing differential equations for a three-fluid heat exchanger are written based on the conservation of energy. The finite element method is used to solve the governing differential equations along with the appropriate boundary conditions. The case of a Buoyonet heat exchanger (used for pasteurizing milk) is analysed and the results are compared with the analytical solution available in the literature. The Buoyonet heat exchanger, treated as a three-fluid heat exchanger is also analysed. The effect of heat loss to the ambient from a parallel flow double pipe heat exchanger is also investigated and the results are compared with those available in the literature. The results are presented both in terms of the temperature distribution along the length of the heat exchanger and the variation of effectiveness with NTU. The methodology presented in this paper can be extended to heat exchangers with any number of streams and any combination of the flow arrangements.

Introduction

Heat exchangers are devices for exchanging energy between two or more fluids. They find applications in various industries like power, process, electronics, refining, cryogenics, chemicals, metals and manufacturing sector. A typical heat transfer mode involves an indirect contact type heat exchanger where heat is transferred by convection from one fluid to a separating wall, through the wall by conduction and then by convection to the other fluid. Even though heat exchanger designs have been reported quite extensively, they are generally limited to steady-state performance, single phase fluids, a few of the many possible flow arrangements and only two fluid heat exchangers. While these designs encompass the majority of the heat exchanger applications, there are some designs, which involve several fluids such as in cryogenics or fault-tolerant heat exchangers (Lalvani *et al.*, 2000). One simple heat exchanger for multi



stream is that of Linde multi-tube heat exchanger used in cryogenic systems (Barron, 1999). In some conventional systems, such as regenerative gas turbine power plants, the system will operate even if the effectiveness of the heat exchanger is less than 50 per cent. In contrast, a cryogenic liquefier will produce no liquid if the heat exchanger effectiveness is less than approximately 85 per cent. Heat exchangers used in cryogenic applications operate at significantly different temperatures from that of the ambient surroundings and, as a result of this temperature difference, there is always some heat exchange between the heat exchanger and its environment. Because of more stringent requirement on effectiveness, environmental heat transfer may result in serious deterioration of the performance of the cryogenic heat exchangers. This case can also be handled as a three-fluid heat exchanger in which the ambient (third fluid) temperature is constant. Hausen (1942) presented an analysis from an earlier work of Nesselmann (1928) which considered thermal losses to ambient. Barron (1984) and Chowdhury and Sarangi (1984a, b) have determined the performance of cryogenic heat exchangers with heat leak from the surroundings.

Many times the multifluid heat exchangers (such as the flat-pin type), especially with phase changes are best simulated by computerized numerical techniques.

A comprehensive treatment of heat exchanger analysis in the presence of a third medium is given by Lalvani *et al.* (2000). Sekulik and Shah (1995) have also discussed in detail the thermal design theory of three-fluid heat exchangers, where they have allowed the third fluid temperature to vary according to the prevailing thermal communications while neglecting interaction with the ambient.

Many of the above-mentioned analyses were carried out with the restriction of constant specific heat. The fluid properties vary considerably in the near critical region and the cryogenic heat exchangers may operate in this region. The specific heat also varies significantly in a condenser in which the fluid enters as a superheated vapour. The heat exchangers surface area determined on the basis of constant specific heats are about 25 per cent of the actual area required! Chowdhury and Sarangi (1984a, b) examined the effect of variable specific heats on the performance of hydrogen heat exchangers whereas, Oonk and Hustvedt (1986) examined the same effect for helium heat exchangers. Soyars (1991) studied the effect of variable fluid properties on the accuracy of analysis of helium heat exchanger performance in the temperature range below 15 K. Their results indicated noticeable errors for helium heat exchangers in refrigeration systems.

Longitudinal conduction along the separating surfaces of the two streams in conventional heat exchangers is often an insignificant effect. For cryogenic heat exchangers, on the other hand, this may result in serious performance deterioration. The effect of longitudinal conduction is most pronounced for heat exchangers having short conduction length (of the order of 100-200 mm) and

large NTU (as high as 500-1,000) as it happens in cryogenic heat exchangers. Kays and London (1984), Kroeger (1967), Landau and Hlinka (1960), Ranganayakulu and Seetharamu (1999a, b) and Soyars (1991) have reported the analysis of longitudinal conduction which are mostly for conventional heat exchangers.

In order to take care of the several factors like effect of ambient, effect of variable fluid properties and the longitudinal heat conduction, for the case of multi stream heat exchangers, it is essential to build-up a systematic approach for determining the performance. Finite element method being versatile in handling both linear and non-linear problems as well as its ability to give the data at element level, is considered in the present analysis.

The governing equations for a three-fluid heat exchanger are being written down based on the conservation of energy. The heat exchanger is being discretised and then the appropriate boundary conditions at the hot and cold fluid inlets are specified. The case of a Buoyonet heat exchanger (used for pasteurizing milk) is analysed and the results are compared with the analytical solution available in the literature. The effect of heat exchange with the ambient is also investigated and the results are compared with the available results in the literature. Some of the flow arrangements, possibly for three-fluid heat exchangers, are also considered in this investigation. The effectiveness of the multifluid heat exchanger is calculated and presented in the graphical form against NTU.

Analysis

The numerical analysis developed is based on the finite element method. FEM utilises discretisation of the domain over which the solution is sought and obtains the characteristic of one element in the form of an element matrix. Figure 1 shows the schematic diagram of a three-fluid heat exchanger discretised into ten elements. The hot fluid 3 loses heat to the cold fluid 2 while the cold fluid 1 gains heat from the cold fluid 2.

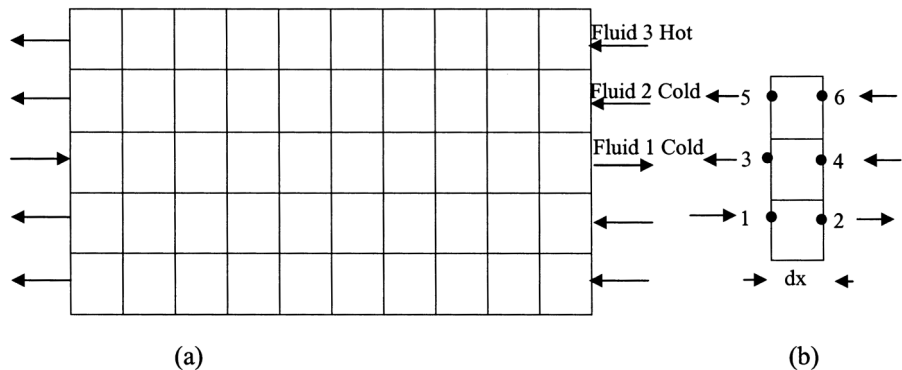


Figure 1.
(a) Three-fluid heat exchanger discretised into ten elements; and
(b) single element with nodal points shown

Discretisation of three-fluid heat exchanger

Figure 1(b) shows a typical six noded element of the three-fluid heat exchanger considered here. There are three nodes at entry and three nodes at the exit.

Governing equations and finite element formulation

The differential equation governing heat transfer from the hot fluid 3 to the cold fluid 2 over an elemental distance dx in terms of temperature T is:

$$C_3 \frac{dT_h}{dx} = -U_{32}(T_h - T_{c2})\pi D_2 \quad (1)$$

where C_3 is the heat capacity rate of the hot fluid, U_{32} the overall heat transfer coefficient for the outer tube, T_{c2} the temperature of the cold fluid 2, T_h the hot fluid temperature, and D_2 the diameter of the heat exchange surface between the fluids 2 and 3.

Similarly, the differential equation governing heat gained by the cold fluid 2 from the hot fluid 3 and the heat loss to cold fluid 1 over an elemental distance dx in terms of temperature T is:

$$C_2 \frac{dT_{c2}}{dx} = \pi D_2 U_{32}(T_h - T_{c2}) - \pi D_1 U_{21}(T_{c2} - T_{c1}) \quad (2)$$

where C_2 is the heat capacity rate of the cold fluid 2, T_{c1} is the temperature of the cold fluid 1 and U_{21} is the overall heat transfer coefficient of the inner tube.

And, the differential equation governing the heat gained by the cold fluid 1 from the cold fluid 2 over an elemental distance dx in terms of temperature T is:

$$C_1 \frac{dT_{c1}}{dx} = U_{21} \pi D_1 (T_{c2} - T_{c1}) \quad (3)$$

where C_1 is the heat capacity rate of the cold fluid 1.

Assuming linear variation for the hot and cold fluids in the element as:

$$T_h = N_1 T_6 + N_2 T_5; \quad T_{c1} = N_1 T_1 + N_2 T_2 \quad (4)$$

$$T_{c2} = N_1 T_4 + N_2 T_3 \quad (5)$$

where the shape functions are given by;

$$N_1 = 1 - \frac{A}{\Delta A} \quad \text{and} \quad N_2 = \frac{A}{\Delta A} \quad (6)$$

A is the area at any given location, which varies from 0 to A and ΔA is the area of an element. Applying this approximation to governing equation (1), the following two equations are obtained:

$$\int N_1 \left[C_3 \frac{dT_h}{dx} + U_{32}(T_h - T_{c2})\pi D_l \right] dx = 0 \quad (7)$$

$$\int N_2 \left[C_3 \frac{dT_h}{dx} + U_{32}(T_h - T_{c2})\pi D_l \right] dx = 0 \quad (8)$$

and minimising the residual error by Galerkin's method, we obtain the following equations:

$$\frac{C_3}{2} [-T_6 + T_5] + M_2 [2T_6 + T_5 - 2T_4 - T_3] = 0 \quad (9)$$

$$\frac{C_3}{2} [-T_6 + T_5] + M_2 [T_6 + 2T_5 - T_4 - 2T_3] = 0 \quad (10)$$

where

$$M_2 = \frac{U_{32}\pi D_2 l}{6} = \frac{U_{32}Ah_c}{6}.$$

Applying the approximations to governing equation (2) and minimising the residual error by Galerkin's method, we obtain the following two equations:

$$\begin{aligned} T_1(-2M_1) + T_2(-M_1) + T_3 \left(\frac{C_2}{2} + M_2 + M_1 \right) \\ + T_4 \left(-\frac{C_2}{2} + 2M_2 + 2M_1 \right) + T_5(-M_2) + T_6(-2M_2) = 0 \end{aligned} \quad (11)$$

$$\begin{aligned} T_1(-M_1) + T_2(-2M_1) + T_3 \left(\frac{C_2}{2} + 2M_2 + 2M_1 \right) \\ + T_4 \left(-\frac{C_2}{2} + M_2 + M_1 \right) + T_5(-2M_2) + T_6(-M_2) = 0 \end{aligned} \quad (12)$$

where,

$$M_1 = \frac{U_{21}\pi D_1 l}{6} = \frac{U_{21}Ah_c}{6}.$$

Similarly, applying the approximations to governing equation (3) and minimising the residual error by Galerkin's method, we obtain the following two equations:

$$T_1 \left(-\frac{C_1}{2} + 2M_1 \right) + T_2 \left(\frac{C_1}{2} + M_1 \right) + T_3(-M_1) + T_4(-2M_1) = 0 \quad (13)$$

$$T_1 \left(-\frac{C_1}{2} + M_1 \right) + T_2 \left(\frac{C_1}{2} + 2M_1 \right) + T_3(-2M_1) + T_4(-M_1) = 0 \quad (14)$$

Equations (9)-(14) form a set of algebraic equations for the unknowns at the six nodes of one element. With the help of these equations a 6×6 element matrix is obtained as follows:

$$\begin{bmatrix} -\frac{C_1}{2} + 2M_1 & -M_1 & 0 & \frac{C_1}{2} + M_1 & -2M_1 & 0 \\ -2M_1 & \frac{C_2}{2} + M_1 + M_2 & -M_2 & -M_1 & -\frac{C_2}{2} + 2M_1 + 2M_2 & -2M_2 \\ 0 & -M_2 & \frac{C_3}{2} + M_2 & 0 & -2M_2 & -\frac{C_3}{2} + 2M_2 \\ -\frac{C_1}{2} + M_1 & -2M_1 & 0 & \frac{C_1}{2} + 2M_1 & -M_1 & 0 \\ -M_1 & \frac{C_2}{2} + 2M_1 + 2M_2 & -2M_2 & -2M_1 & -\frac{C_2}{2} + M_1 + M_2 & -M_2 \\ 0 & -2M_2 & \frac{C_3}{2} + 2M_2 & 0 & -M_2 & -\frac{C_3}{2} + M_2 \end{bmatrix} \times \begin{pmatrix} T_1 \\ T_3 \\ T_5 \\ T_2 \\ T_4 \\ T_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (15)$$

Global matrix is obtained after assembling the element matrix for all the elements considered. Boundary conditions are imposed and thereafter the global matrix is solved to obtain the temperature distribution along the heat exchanger.

Buoyonet heat exchanger

The case of a Buoyonet heat exchanger is considered here for the purpose of validation of the methodology used in view of the availability of the analytical solution for this case. Figure 2(a) shows the arrangement of a Buoyonet heat exchanger (discretised into ten elements) where cold fluid 1, after its exit from

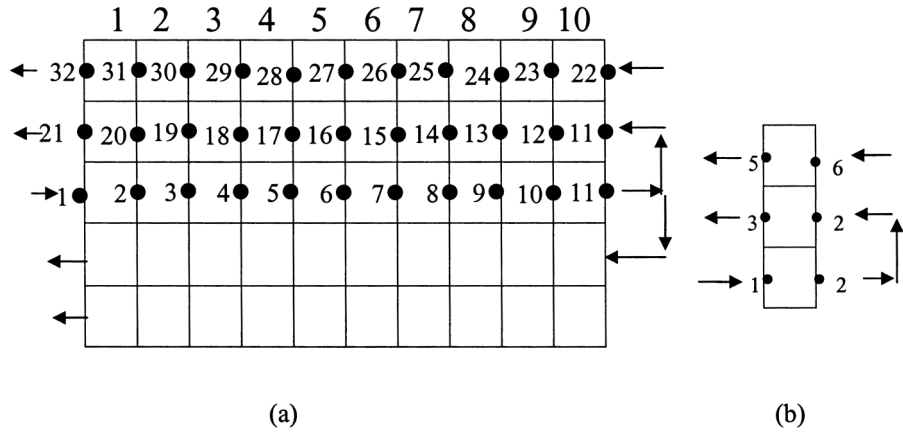


Figure 2.
(a) Buoyonet heat exchanger with ten discretised elements; and
(b) one typical element with nodal points

the heat exchanger, is allowed to flow into the passage for cold fluid 2 of the three-fluid heat exchanger. Thus, cold fluid 2 gains further heat from hot fluid 1 as it traverses along the heat exchanger as shown in Figure 2(a). Figure 2(b) shows an element with its nodal points. Following the same method as explained for the three-fluid heat exchanger, the element matrix for the last element of the Buoyonet heat exchanger, which has five nodes, as shown in Figure 2(b), is given as:

$$\begin{bmatrix} -\frac{C_1}{2} + 2M_1 & -M_1 & 0 & \frac{C_1}{2} - M_1 & 0 \\ -2M_1 & \frac{C_2}{2} + M_1 + M_2 & -M_2 & -\frac{C_2}{2} + M_1 + M_2 & -2M_2 \\ 0 & -M_2 & -\frac{C_3}{2} + M_2 & -2M_2 & -\frac{C_3}{2} + 2M_2 \\ -\frac{C_1}{2} + M_1 & -2M_1 & 0 & \frac{C_1}{2} + M_1 & 0 \\ 0 & -2M_2 & \frac{C_3}{2} + 2M_2 & -M_2 & -\frac{C_3}{2} + M_2 \end{bmatrix}$$

$$\times \begin{Bmatrix} T_1 \\ T_3 \\ T_5 \\ T_2 \\ T_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (16)$$

The global matrix is obtained after assembling all the element matrices. The solution of the global matrix is carried out after imposing the boundary conditions to obtain the temperature distribution of the hot and cold fluids at different nodal points of the discretised heat exchanger.

Three-fluid heat exchanger with one fluid as ambient

This type of arrangement is similar to the case of a double pipe heat exchanger except that there is a heat transfer from the fluid flowing in the outer pipe to the ambient. Considering hot fluid to flow in the inner pipe parallel to the cold fluid flow direction and the ambient temperature kept constant as T_∞ , the element matrix in this case is obtained as:

$$\begin{pmatrix}
 -\frac{C_H}{2} + M1 & -M1 & \frac{C_H}{2} + 2M1 & -2M1 \\
 -2M1 & -\frac{C_C}{2} + 2M1 + 2M2 & -M1 & \frac{C_C}{2} + M1 + M2 \\
 -\frac{C_H}{2} + 2M1 & -2M1 & \frac{C_H}{2} + M1 & -M1 \\
 -M1 & -\frac{C_C}{2} + M1 + M2 & -2M1 & \frac{C_C}{2} + 2M1 + 2M2
 \end{pmatrix}
 \times
 \begin{pmatrix}
 T_1 \\
 T_3 \\
 T_2 \\
 T_4
 \end{pmatrix}
 =
 \begin{pmatrix}
 0 \\
 3M2T_\infty \\
 3M2T_\infty \\
 0
 \end{pmatrix}
 \tag{17}$$

where C_H and C_C are the heat capacity rates of the hot and cold fluid, respectively, $M1 = U_1(\Delta A)_I/6$, $M2 = U_0(\Delta A)_O/6$, and U_1 and U_0 are the overall heat transfer coefficients for the inner and outer tubes, respectively.

Similarly, when the cold fluid flows in the inner tube and its direction is parallel to the hot fluid flowing in the outer tube, there will be heat transfer from the outer tube to the ambient. Following the same method as explained earlier, the element matrix under this case is obtained as:

$$\begin{pmatrix}
 -\frac{C_c}{2} + 2M1 & -2M1 & \frac{C_c}{2} + M1 & -M1 \\
 -2M1 & -\frac{C_H}{2} + 2M1 + 2M2 & -M1 & \frac{C_H}{2} + M1 + M2 \\
 -\frac{C_c}{2} + M1 & -M1 & -\frac{C_c}{2} + 2M1 & -2M1 \\
 -M1 & -\frac{C_H}{2} + M1 + M2 & -2M1 & \frac{C_H}{2} + 2M1 + 2M2
 \end{pmatrix}
 \times
 \begin{pmatrix}
 T_1 \\
 T_3 \\
 T_2 \\
 T_4
 \end{pmatrix}
 =
 \begin{pmatrix}
 0 \\
 3M2T_\infty \\
 3M2T_\infty \\
 0
 \end{pmatrix}
 \tag{18}$$

As mentioned earlier, for the above two cases of parallel flow heat exchanger when third fluid is ambient, the global matrix is obtained after the assembly of the element matrices. The solution of the global matrix is obtained after applying the known boundary conditions.

Results and discussion

The methodology described earlier is applied first to Buoyonet heat exchanger to determine the temperature distribution at different node points and compare the results with those available in the literature. Next, three-fluid heat exchanger is analysed by considering the cold fluid 2 (as a separate fluid) to enter the tube at the same temperature as that of the exit of the cold fluid 1. Then, the heat exchanger with the third fluid as ambient is analysed (in parallel mode) and the results are compared with those available in the literature. All these exercises show the accuracy and validity of the methodology used to analyse the multifluid heat exchangers to determine their performances.

The following fluid properties for the analysis of the Buoyonet heat exchanger are considered.

- Inner heat transfer coefficient, $U_{21} = 341.5 \text{ W/m}^2 \text{ K}$
- Outer heat transfer coefficient, $U_{32} = 683.00 \text{ W/m}^2 \text{ K}$
- Heat capacity rate of cold fluid, $C_1 = C_2 = 264.6 \text{ W/}^\circ\text{C}$
- Heat capacity rate of hot fluid, $C_3 = 132.3 \text{ W/}^\circ\text{C}$
- Inlet temperature of hot fluid, $T_h = 93.3^\circ\text{C}$
- Inlet temperature of cold fluid 1, $T_{c1} = 10.0^\circ\text{C}$
- Inlet temperature of cold fluid 2, $T_{c2} = 18.798^\circ\text{C}$
- Outer elemental heat transfer area, $A_{hc} = 0.0486 \text{ m}^2$

- Inner elemental heat transfer area, $A_{cc} = 0.0243 \text{ m}^2$
- Number of elements = 10

The calculated temperature distribution at different nodal points using the present method is shown in Table I. The comparison with the analytical results of Kern (1972) and with those of Kumaur (1988) is satisfactory. This validates the accuracy of the present methodology.

Next, the three-fluid heat exchanger is analysed considering three different fluids with their inlet temperatures known. In this case, the inlet temperature for the cold fluid 2 is taken equal to the outlet temperature of the cold fluid 1 from the previous analysis carried out. Solution of the global matrix for the three-fluid heat exchanger with the above boundary conditions and same fluid properties as given for Buoyonet heat exchanger gives the same result of the temperature distribution at different nodal points as expected. Thus, the accuracy of the methodology adopted for the analysis of the three-fluid heat exchanger is verified.

Next, the three-fluid heat exchanger is analysed with one of the fluids as ambient which is considered to be at constant temperature and the mode of the fluid flow directions being parallel. The hot fluid is flowing in the inner tube. The following fluid properties for the analysis of such a heat exchanger is considered. It may be noted that the analysis is carried out for different values of the outer heat transfer coefficients as well.

- Inner heat transfer coefficient, $U_I = 500 \text{ W/m}^2 \text{ K}$
- Outer heat transfer coefficient, $U_O = 0.0, 0.25 U_I, U_I$
- Heat capacity rate of cold fluid, $C_C = 500 \text{ W}$
- Heat capacity rate of hot fluid, $C_H = 2,000 \text{ W}$
- Length of heat exchanger, $L = 3 \text{ m}$
- Inlet temperature of hot fluid = 100°C
- Inlet temperature of cold fluid = 40.0°C

Node number	Kern (1972) ($^\circ\text{C}$)	Kumaur (1988) ($^\circ\text{C}$)	Present method ($^\circ\text{C}$)
1	10.0	10.0	10.000
5	14.9	15.0	15.029
10	18.6	18.6	18.621
15	28.4	28.5	28.527
20	32.75	32.8	32.841
21	32.6	32.7	32.734
23	93.3	93.3	93.300
25	77.6	77.6	77.595
27	66.7	66.6	66.642
29	58.8	58.8	58.794
31	53.0	53.0	52.969

Table I.
Temperature
distribution

- Ambient temperature = 20.0°C
- Diameter of inner pipe = 0.5 m
- Diameter of outer pipe = 1.0 m

The calculated temperature distribution of the different fluids (in this case, hot and cold fluid) along the length of the heat exchanger for different values of the heat capacity ratio is shown in Figures 3-5. These figures also show the analytical results of the above heat exchanger in parallel mode of action as given by Lalvani *et al.* (2000). It is observed that the temperature distribution obtained by the present finite element method using Galerkin's technique for minimizing the residual errors is very close to that given by the analytical

Figure 3.
Temperature variation along the length of the heat exchanger for $C_C/C_H = 0.25$ and $U_O = 0.0$ (hot fluid inside)

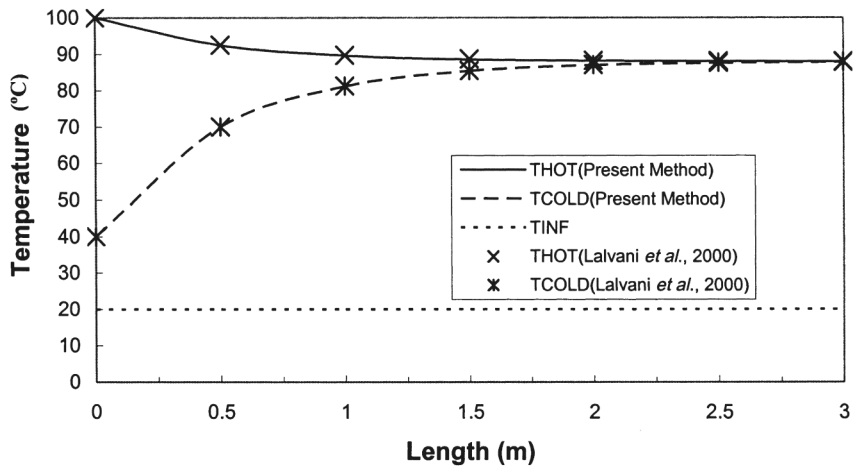
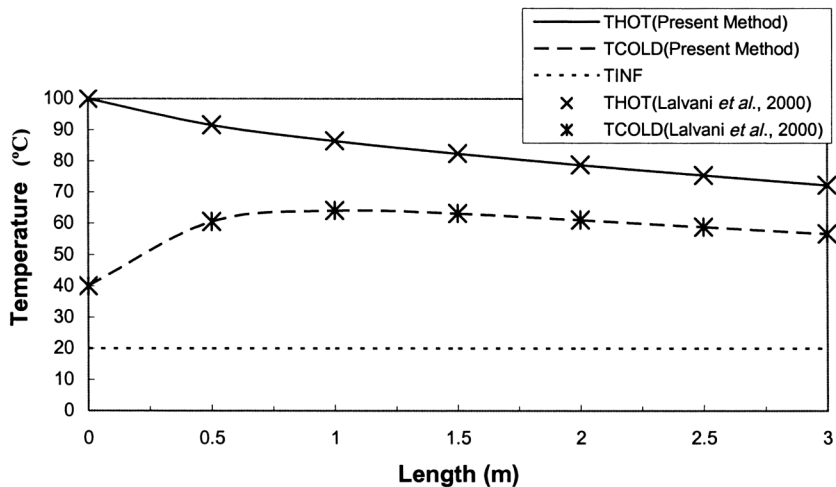


Figure 4.
Temperature variation along the length of the heat exchanger for $C_C/C_H = 0.25$ and $U_O = 0.25 U_I$ (hot fluid inside)



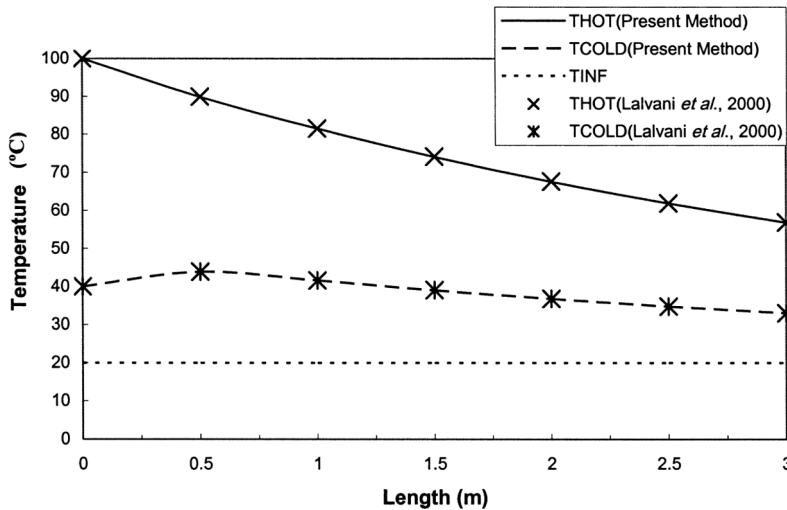


Figure 5. Temperature variation along the heat exchanger length for $C_C/C_H = 0.25$ and $U_O = U_I$ (hot fluid inside)

solution (Lalvani *et al.*, 2000). Thus, the accuracy of the present method for the sensible heat transfer region is again demonstrated and validated.

Figure 6 also shows the temperature variation of the two streams as considered earlier with the difference that the cold fluid flows inside. In this case, the cold fluid outlet temperature exceeds that of the hot fluid. Also, it can be seen from Figure 6 that the temperature of the cold fluid increases first up to about 0.75 m and then starts decreasing for the example considered.

Figure 7 shows a plot of effectiveness against NTU for a three-fluid heat exchanger in which the outermost fluid is ambient whose temperature is considered to be constant. The heat losses to the ambient depend upon the overall heat transfer coefficient U_O . The case $U_O = 0.0$ corresponds to the conventional double pipe parallel flow heat exchanger with no heat losses. Different levels of heat losses to the ambient are also shown in Figure 7. It is

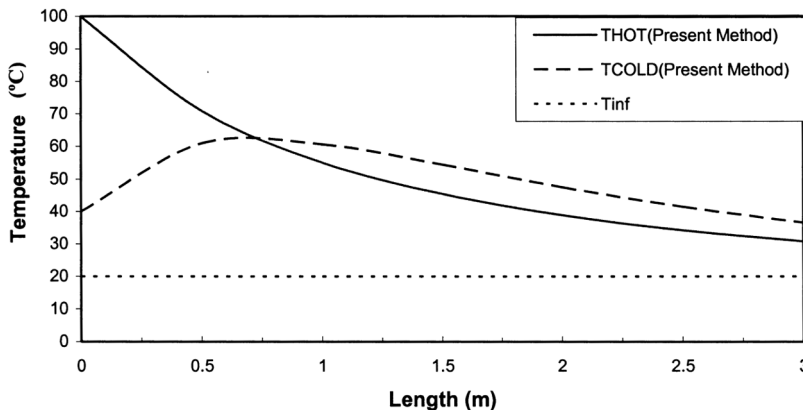
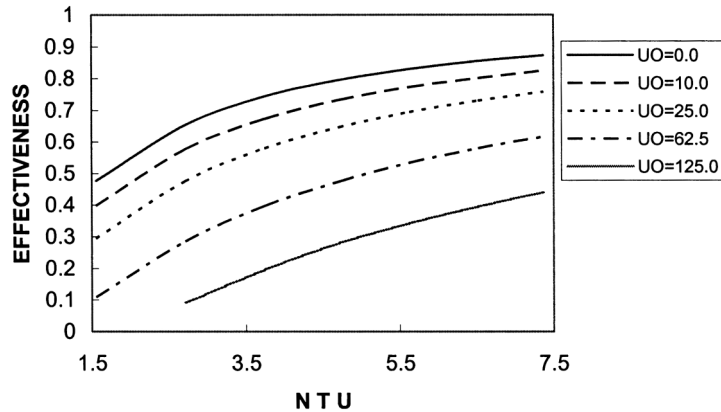


Figure 6. Temperature variation along the heat exchanger length for $C_C/C_H = 0.25$ and $U_O = U_I$ (cold fluid inside)

Figure 7.
Performance of heat exchanger with different heat losses to the ambient parallel flow with hot fluid inside



observed from Figure 7 that by increasing the value of U_o , the effectiveness decreases for the same NTU, as expected.

For a three-fluid heat exchanger, one can follow the same procedure as explained by Sekulik and Shah (1995) to express its overall effectiveness in terms of NTU_1 (the thermal size of a heat exchanger). The value of NTU_1 represents the ability of the heat exchanger to change the temperature of fluid 1 because of the thermal communication between streams 1 and 2 (Chato *et al.*, 1971).

Conclusions

Application of the finite element method to analyse a three-fluid heat exchanger is illustrated through an example of Buoyonet heat exchanger. The heat losses from a two-fluid heat exchanger to the ambient is also investigated as another example of the three-fluid heat exchanger. From the analyses, the following conclusions are drawn.

- (1) The performance of the Buoyonet heat exchanger predicted by using the finite element method compares well with the analytical solution.
- (2) In the case of a double pipe heat exchanger with cold fluid flowing in the inner tube, the effect of the ambient on the temperature distribution shows a possibility of temperature crossover. In such a case, in some portion of the heat exchanger, the cold fluid is at a higher temperature compared to the hot fluid.
- (3) The reduction in the effectiveness of the heat exchanger due to heat losses to the ambient is clearly brought out through the analysis of a three-fluid heat exchanger.
- (4) The variation in the heat transfer coefficient along the length of the heat exchanger for each fluid stream can be easily accounted in the present methodology in view of the heat exchanger being treated as an assemblage of elements.

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